Fig. 2. In marked contrast to the uniformly good results obtained for skin friction, the refined integral method predicts a displacement thickness distribution that diverges drastically from the rapid rise of the exact solution when the skin friction becomes small (incipient blow-off), tending in fact toward the linear behavior given by the ordinary Kármán-Pohlhausen and Rayleigh analogy approximations. Evidently, δ^* is quite sensitive to the skin-friction behavior near blow-off. It is also noteworthy from Fig. 2 that the local similarity approximation once again yields qualitatively correct results. Clearly, then, in spite of its generally good accuracy in predicting skin friction, the refined moment integral approach still possesses some deficiencies that would limit its usefulness in treating strong blowing and incipient separation problems where an accurate account of the displacement effects is essential.

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Reply by Author to G. R. Inger

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THE author wishes to thank Professor Inger for taking interest in the author's work.[1] His enthusiastic efforts in exploring the potential of the approximate method, currently being investigated by the author, are particularly appreciated. Stimulated by Inger's comments, the author feels that some further remarks about Ref. 1 are in order.

The nature and basic ideas of this approximate method along with its principal merits have been elucidated on several occasions in the *AIAA Journal* [see Ref. (2) in particular], and, therefore, require no further elaboration.

The merits of the present method are highlighted in Fig. 1 of the Comment which illustrates an interesting comparison on the results produced by a wide variety of approximate methods of different nature. In response to Inger's remarks on the obvious inadequacy of the method near blow-off, it is perhaps appropriate to reiterate here that the present method is only meant to be a refinement of the usual Kârmân-Pohlhausen (K-P) method. Being aware of the approximate nature of the method the present author certainly makes no claim as to its perfection. Obviously, Ref. 1 presented the method as a simple, practical tool for studying usual boundary-layer flows with surface mass transfer. In this form, it was never intended for studying the delicate and difficult problem of blow-off, separation or separated flows.

In light of the aforementioned discussion, Fig. 2 of the Comment provides further evidence to the accuracy of the present method. It is emphasized that due care must be exercised in observing the obvious region of its intended applications, i.e., $\varepsilon^2 Re_x < (\varepsilon^2 Re_x)_{\tau_w=0}$ (≈ 0.4 for the particular profile under discussion; $\varepsilon \equiv v_w/u_\infty$). Accordingly, Inger's calculation of δ^*, based on the solutions of Ref. (1), should have been limited to this region where the refined integral method [Ref. (1)] predicts positive skin friction. The importance of the effective displacement thickness in studying the problem of viscous-inviscid interaction in the presence of surface mass transfer has received considerable attention from most fluid dynamicists, including the present author[3] (unsteady, weak interactions). Interested readers are referred to Fannelop[4] and Li[5] among others, for a more thorough discussion on the subject and pertinent references. We only note that the conventional displacement thickness, δ^*, alone generally does not represent the total displacement effect. Modifications should be made to account for the effects of surface mass transfer.

In closing, the author is of the opinion that the merits of the refined K-P method, as illustrated and reiterated in Refs. 1 and 2, are best exploited when applications are made to the calculations of surface properties. The extension of the method to study problems of massive blowing, viscous interaction, or separated flows must proceed with caution.

Finally, the author would like to take this opportunity to correct a minor typographical error in Ref. 1. The suction strength in the caption of Fig. 4 should read $\varepsilon A^{1/2} = -1/(2)^{1/2}$.

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Index category: Boundary Layers and Convective Heat Transfer—Laminar.

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Comment on "Finite Elements for Axisymmetric Solids under Arbitrary Loadings with Nodes on Origin"

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IN a Technical Note, Belytschko[1] outlines how a finite element analysis of axisymmetric solids using linear displacement triangular ring elements may correctly include nodes lying on the

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